

Optimization of Repetition Parameters For Threshold Scanning

C. Flacco, A. Ciocio, V. Fadeyev
ATLAS SCT Group, LBNL

July 2, 2001

Abstract

Through statistical analysis and error propagation, we study the relationships between precision in noise, gain, offset, and trim slope; and the parameters of repetition. These parameters are number of voltage steps in each scan, N ; number of triggers per voltage point, N_{evts} ; and number of scan points, n . The noise precision provides a minimum condition for the product $N(N_{evts} - 1)$. The constraints on scan points given by offset and trim slope precisions are also given.

1. Introduction

To measure the parameters of gain, offset, noise, and trim slope of the ABCD3T chips used in the ATLAS SCT detector, we use the method of threshold scanning [1]. Each threshold scan produces an s-curve (Figure 1) with a transition width, σ , that characterizes the noise in the system. The mid-point of this transition is referred to as the 50% point. By scanning at different charge-injection points, we generate a response curve (Figure 2) for each channel of the chip. In this way we use the measured 50% points to extract the gain and offset. This method is necessitated by the use of binary readout in the ABCD chip design [2].

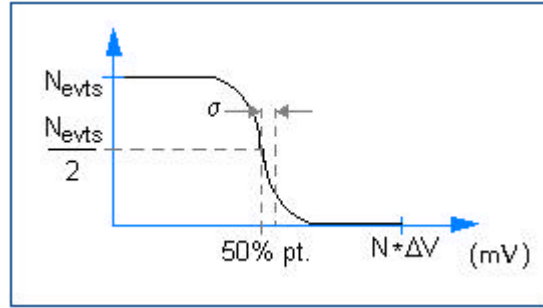


Figure 1. Illustration of an s-curve from a threshold scan occupancy histogram.

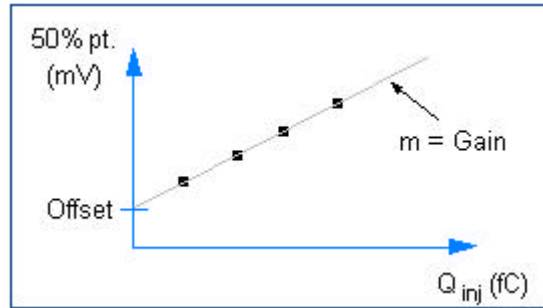


Figure 2. Illustration of a response curve. Fifty-percent point is plotted as a function of trigger pulse height. The gain and offset of the channel are the slope and intercept, respectively, of the best-fit line for the response curve.

Currently, we are generating 500 events ($N_{evts} = 500$) with $N = 60$ equal threshold voltage steps between $V_{LO} = 50\text{mV}$ and $V_{HI} = 500\text{mV}$ ($DV = 7.5\text{mV}$) to generate occupancy histograms for the ABCD3T chips. We scan at n different pulse amplitudes. The current default value of n is four. We would like to understand how the statistical uncertainties in noise, gain, offset depend on these three repetition parameters. We also use threshold scanning to extract the trim slope of each channel [1], and will apply the present analysis to this quantity also.

2. Precision in Noise

The optimal values of the repetition parameters are constrained by the requisite level of uncertainty in the noise, \mathbf{s} . The transition we are measuring has the shape of a scaled complementary error function, $erfc()$. We require an equation for the statistical error of this distribution. Our experimental constraints force us to consider only binned data, so the relation must also account for smearing effects.

We obtain such a relation using the Monte Carlo technique. This has two primary advantages: first, it allows us to study the smearing effects mentioned above; second, it permits us to understand and quantify the situation $\mathbf{s} / DV \ll 1$. The latter is important here because the current default values approach this regime. The function $\mathbf{ds}(\mathbf{s}, N_{evts}, DV)$ in the standard regime obtained from this study is given here. For a description of the Monte Carlo study, see Appendix A.

We find the following relation for the usual case of $\mathbf{s} / DV \gg 1$:

$$\mathbf{ds}(\mathbf{s}, N_{evts}, \Delta V) = \sqrt{\frac{\mathbf{s} * \Delta V}{(N_{evts} - 1)}}$$

We can also express this in terms of the number of voltage steps, N :

$$\mathbf{ds} = \sqrt{\frac{\mathbf{s}(V_{HI} - V_{LO})}{N(N_{evts} - 1)}}$$

By imposing a bound, f_s , on fractional error in the measured values of noise, \mathbf{s} , we also bound \mathbf{ds} . We obtain the minimum condition

$$N(N_{evts} - 1) = \frac{V_{HI} - V_{LO}}{\mathbf{s} * f_s^2}$$

3. Precision in Gain and Offset

The optimization must also consider constraints from the precision in gain and offset. The relationship between the gain, offset and 50% point is

$$(50\% \text{ pt}) = (Gain) * Q_{inj} + (Offset)$$

The parameter N_{evts} enters the uncertainties in gain and offset through the uncertainty in the 50% point of the occupancy histogram. Specifically, the uncertainty in the 50% point has the following dependence on N_{evts} [3]:

$$\mathbf{d}(50\% pt) = \mathbf{s} \sqrt{\frac{r}{N_{evts} \sqrt{\mathbf{p}}}} + \mathbf{m}^2(r)$$

Here, \mathbf{s} is the width of the transition in the s-curve. If we define $\mathbf{DV} = (V_{HI} - V_{LO}) / N$, the parameter r is defined as \mathbf{DV}/\mathbf{s} , and $\mathbf{m}(r)$ is a documented function with values $0 < \mathbf{m} < 0.004$ for $r < 2$ [3]. Since r is well within this interval in our case, the second term is negligible, and by substituting for r we obtain this relationship for the dependence of $\mathbf{d}(50\% pt)$ on N_{evts} :

$$\mathbf{d}(50\% pt) = \sqrt{\frac{\Delta V \mathbf{s}}{N_{evts} \sqrt{\mathbf{p}}}}$$

The gain and offset are derived from the best-fit line for the 50% points at the selected values of Q_{inj} . We can calculate the statistical errors in the gain and offset using standard techniques of error propagation [4].

For linear data with best-fit line $y = mx + b$, there is a coordinate system (x', y') such that the errors on m' and b' are uncorrelated. This is the frame in which $m' = m$, and the intercept b' is the height of the line at $\langle x \rangle$, the centroid value of x .

The uncertainty in the slope, m , of such a line is given by:

$$\mathbf{d}m = \mathbf{d}m' = \sqrt{\frac{1}{\sum_{i=1}^n (x'_i / \mathbf{d}_i)^2}}$$

In this relation and subsequently, n is the number of points available for the fit.

The uncertainty of the intercept in the primed coordinates is given below.

$$\mathbf{d}b' = \mathbf{d}\langle y \rangle = \sqrt{\frac{1}{\sum_{i=1}^n (1/\mathbf{d}_i)^2}}$$

The uncertainty in any y' -value at a given x' -value is given by the sum of the uncertainties in the slope (weighted by position x') and intercept, added in quadrature, where the uncertainty in the intercept is given by the relation below.

$$\mathbf{d}b = \sqrt{\langle x \rangle^2 \mathbf{d}m'^2 + \mathbf{d}b'^2}$$

We will use these abstract relations to create specific equations in the physical variables of our experiment. The correspondence is $x \rightarrow Q_{inj}$, $y \rightarrow 50\% pt.$, $m \rightarrow Gain$, $b \rightarrow Offset$, and $b' \rightarrow <50\% pt.>$. Thus we have these two equations:

$$\mathbf{d}(Gain) = \sqrt{\frac{1}{\sum_{i=1}^n \left((Q_{inj})'_i / \mathbf{d}(50\% pt_i) \right)^2}}$$

$$\mathbf{d}(Offset) = \sqrt{<Q_{inj}>^2 \mathbf{d}(Gain)^2 + \mathbf{d}(<50\% pt.>)^2}$$

We have as well an intermediate one,

$$\mathbf{d}(<50\% pt.>) = \sqrt{\frac{1}{\sum_{i=1}^n (1/\mathbf{d}(50\% pt_i))^2}}$$

By imposing fractional errors, f_G and f_{off} , on the measured values of gain and offset, we bound also $\mathbf{d}(Gain)$ and $\mathbf{d}(Offset)$. We construct the following minimum conditions on the repetition parameters:

$$N_{evts} N * \sum_{i=1}^n (Q_{inj})'_i{}^2 = \frac{s(V_{HI} - V_{LO})}{\sqrt{p} f_G^2 (Gain)^2}$$

$$\frac{1}{N * N_{evts}} \left(\frac{<Q_{inj}>^2}{\sum_{i=1}^n (Q_{inj})'_i{}^2} + \frac{1}{n} \right) = f_{off}^2 (Offset)^2$$

To produce the simplified results above, we have noted that $\mathbf{d}(50\% pt)_i = \mathbf{d}(50\% pt)$ for all values of i . This is the statement that the noise in the system is not dependent on the amount of charge injected, which is true in the regime scanned. It is useful to note that this inequality has the expected dependence on N and N_{evts} . The dependence on n is complicated because our choice of scan points affects the quality of our fitting parameters. We note that the dependence on any one of the three parameters is factorable from the dependence on the other two.

4. Precision in Trim Slope

Our current default is to use the same values for N_{evts} and DV to generate s-curves for calculating trim slope, but here the scan points represent different trim values rather than different pulse heights. This may not be preferable for the following reason. The trim slope has an error that follows the same analysis as that for the gain. The only difference is in the coordinates of the x -axis, and the locations of the x_i . Because we are not concerned with the intercept in this fit, we

do not need the level of precision here that we need for the previous calculation. The minimum condition for the trim slope is

$$N_{evts} N * \sum_{i=1}^n (Trim)_i'^2 = \frac{s(V_{HI} - V_{LO})}{\sqrt{P} f_{TS}^2 (Trim_Slope)^2}$$

5. Analysis of Two Special Cases

There are a couple of interesting special cases to consider which lead to additional inequalities that we may use to optimize our choice of scan points.

The case in which the same precision is needed on all quantities provides a guideline for choosing charge injection scan points. Since we must use sufficient statistics for the above methods to be sound anyway, we consider the approximation $(N_{evts} - 1) \approx N_{evts}$. In this case, we combine the constraints from gain offset and noise. In the case $f_G = f_{off}$, the offset is the constraining parameter. If we now substitute the bound provided by the noise, we are left with a minimum condition in the quantities of the scan points:

$$\left(\frac{\langle Q_{inj} \rangle^2}{\sum_{i=1}^n (Q_{inj})_i'^2} + \frac{1}{n} \right) = \frac{(Offset)^2 (V_{HI} - V_{LO})}{s}$$

In the second case, we consider a constraint on the trim scan points. With the restriction that N_{evts} and N must have the same values in the trim scans that they have in scans measuring gain and offset, the condition from the trim slope precision can be manipulated to emphasize the scan points:

$$\sum_{i=1}^n (Trim)_i'^2 = \frac{s(V_{HI} - V_{LO})}{\sqrt{P} N_{evts} N * f^2 (Trim_Slope)^2}$$

6. Conclusions

We have established the dependence of the precision of the measured values of noise, gain, offset, and trim slope on the repetition parameters N_{evts} , N and n , defined above, using analytic and Monte Carlo techniques. We arrive at four minimum conditions in these parameters.

From the noise, we have,

$$N(N_{evts} - 1) = \frac{V_{HI} - V_{LO}}{s * f_s^2}$$

and from gain and offset we have,

$$N_{evts} N * \sum_{i=1}^n (Q_{inj})_i'^2 = \frac{s(V_{HI} - V_{LO})}{\sqrt{P} f_G^2 (Gain)^2}$$

$$\frac{1}{N * N_{evts}} \left(\frac{\langle Q_{inj} \rangle^2}{\sum_{i=1}^n (Q_{inj})_i^2} + \frac{1}{n} \right) = f_{off}^2 (Offset)^2$$

while the trim slope gives us

$$N_{evts} N * \sum_{i=1}^n (Trim)_i'^2 = \frac{\mathbf{s}(V_{HI} - V_{LO})}{\sqrt{\mathbf{p}} f_{TS}^2 (Trim_Slope)^2}$$

Analysis of the trim slope precision suggests that we may reduce scanning time by using distinct values for N_{evts} and N from those used in measuring gain and offset.

For tables of the typical values of quantities discussed herein and sample precision calculations, see Appendix B.

Appendix A.

Here we describe the method used to deduce the functional form of $\delta\sigma$. Through consideration of related distributions, we expect $\delta\sigma = \delta\sigma(s, N_{evts}, DV)$. We investigate the form of the dependence on each of these variables through the method of Monte Carlo simulation.

To generate a sample of simulated data distributed with s-curve shape, we use the Von Neumann Acceptance-Rejection method [5]. The comparison function used is

$$f(x) = \frac{1}{2} \operatorname{erfc} \left(\frac{x - x_0}{\sqrt{2} s_0} \right)$$

Here, x_0 is the Monte Carlo truth 50% point, and s_0 is the Monte Carlo truth transition width. The random number generator used is the ROOT TRandom::Rndm() function [6].

To deduce the form of $\delta\sigma$, we make histograms of $s - s_0$, where s is the fitted width of a simulated s-curve. The simulated s-curves are fit to the form of a three-parameter complementary error function. The histograms of $s - s_0$ are fit with a three-parameter Gaussian function. The width of such a Gaussian is $\delta\sigma$.

We generate simulated data by separately varying each of the independent variables in $\delta\sigma$. The data generated is given below (Figures 3, 4, 5).

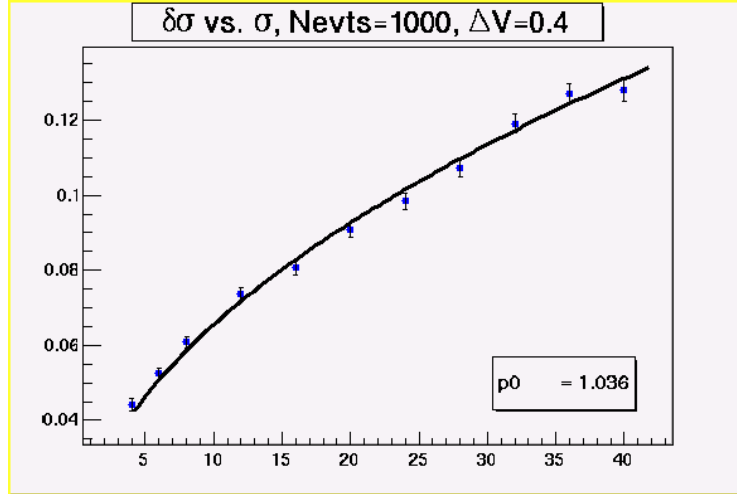


Figure 3. Results of Monte Carlo simulation. $\delta\sigma$ is plotted as a function of s , and fit in the manner described below.

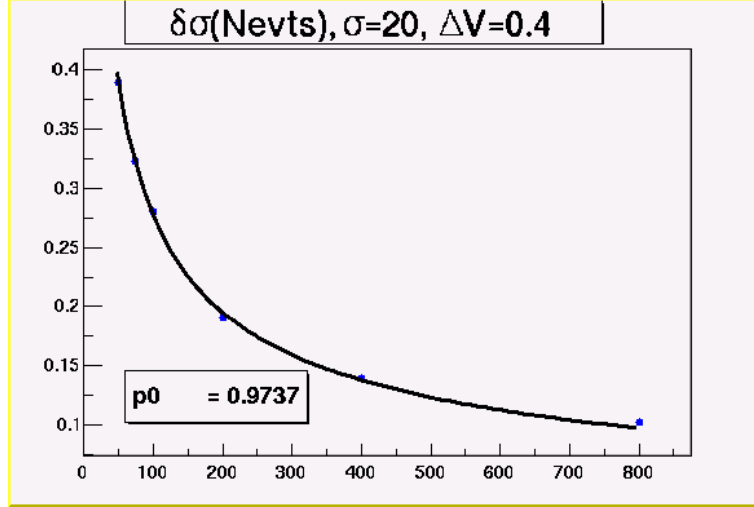


Figure 4. Results of Monte Carlo simulation. $\delta\sigma$ is plotted as a function of N_{evts} , and fit in the manner described below.

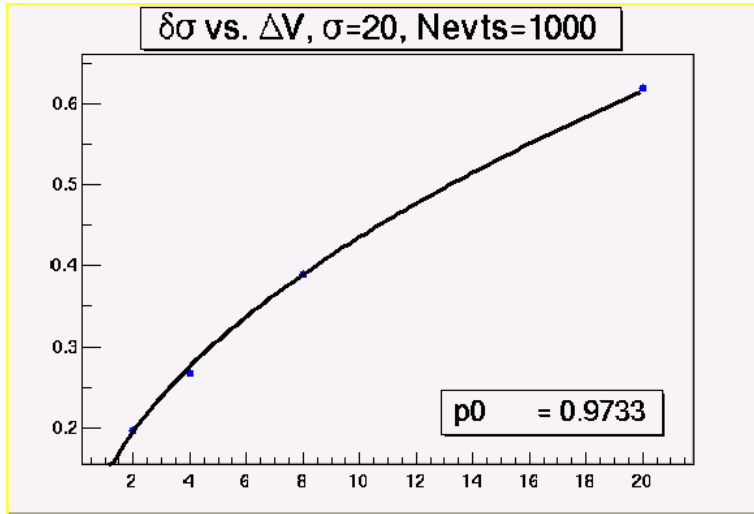


Figure 5. Results of Monte Carlo simulation. $\delta\sigma$ is plotted as a function of ΔV , and fit in the manner described below.

To generate the plots in the figures, we have first fit for the exponent on the independent variable, and then fit with the exponent held constant to deduce the best value for the scaling parameter, p_0 . As the figures illustrate, the most consistent reasonable value for the scaling parameter is $p_0 = 1$.

Combining the results from the three independent variables, we arrive at an equation for $\delta\sigma$:

$$\delta\sigma(s, N_{evts}, \Delta V) = \sqrt{\frac{s * \Delta V}{(N_{evts} - 1)}}$$

Appendix B.

Here are some sample calculations. The values used for the input parameters are currently the default running values for these parameters.

Noise (s) ~ mV	5	Scan points ~ fC
Voltage Step (DV) ~ mV	7.5	
n (# of scan points)	4	
Gain ~ mV/fC	50	
Offset ~ mV	15	
Trim Slope	3	
Nevts	500	
		2.5
		3.0
		3.5
		4.0

The percent statistical errors for various numbers of trigger events are in the tables below.

DV = 7.5mV (default):

Nevts	%c(Gain)	%c(Offset)	%c(Noise)	%c(Trim Slope)
100	0.823	9.046	12.309	1.345
200	0.582	6.396	8.682	0.951
300	0.475	5.223	7.083	0.776
400	0.411	4.523	6.131	0.672
500	0.368	4.045	5.483	0.601
600	0.336	3.693	5.004	0.549
700	0.311	3.419	4.632	0.508

DV = 5.0mV:

Nevts	%c(Gain)	%c(Offset)	%c(Noise)	%c(Trim Slope)
100	0.672	5.539	10.050	1.098
200	0.475	3.917	7.089	0.776
300	0.388	3.198	5.783	0.634
400	0.336	2.770	5.006	0.549
500	0.300	2.477	4.477	0.491
600	0.274	2.261	4.086	0.448
700	0.254	2.094	3.782	0.415

DV = 2.5mV:

Nevts	%c(Gain)	%c(Offset)	%c(Noise)	%c(Trim Slope)
100	0.475	3.917	7.107	0.776
200	0.336	2.770	5.013	0.549
300	0.274	2.261	4.089	0.448
400	0.238	1.958	3.540	0.388
500	0.212	1.752	3.165	0.347
600	0.194	1.599	2.889	0.317
700	0.180	1.480	2.675	0.293

The percent errors in the tables above are given as characteristic only, as they rely upon values of the respective parameters as given in the table above.

Appendix C.

We have performed an experimental verification of the formulae given in the body of this note for the statistical precision in noise, gain and offset.

The plots in Figures 7-12 below compare the theoretical results predicted by our models with the spread of measured values seen in real data. In the real data, each measurement was taken 25 times. Error bars are calculated from statistics and represent 1σ interval.

For the gain and offset, these plots indicate that the theory is consistent with the data. The plots for the noise suggest that the theory is valid in the regime where $r = \Delta V/\sigma \ll 1$, but that the default value for ΔV approaches the outside boundary of this regime. In particular, this is illustrated in Figure 8, where the relative positions of the data and the model become inverted as r is increased. The results in Figure 7, with data taken at $r = 1.5$ (the default), are also consistent. Monte Carlo data in this region is given in Figure 6. The model begins to deviate from the simulated data when r is increased beyond 1.

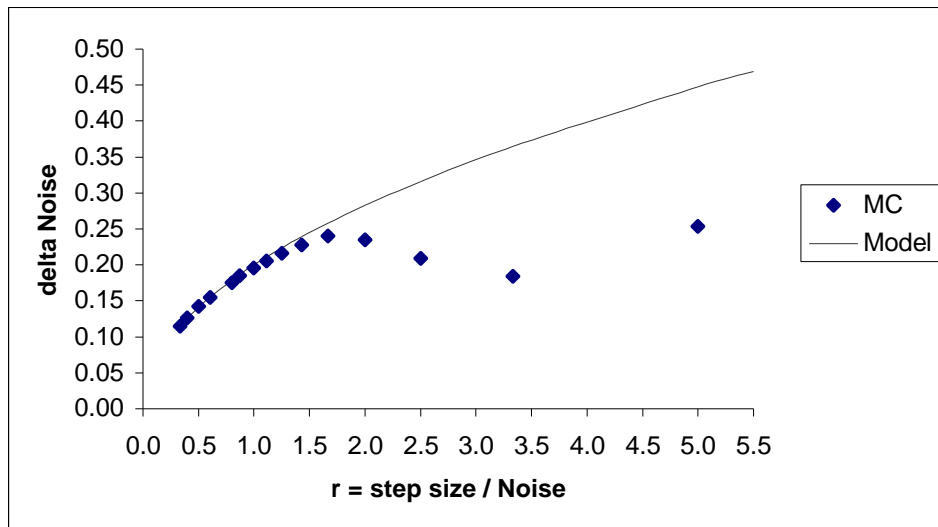


Figure 6. Monte Carlo data showing behavior of $\delta\sigma$ as a function of r . The current default settings have $r=1.5$. Available values below the default are $r=1$ and $r=0.5$.

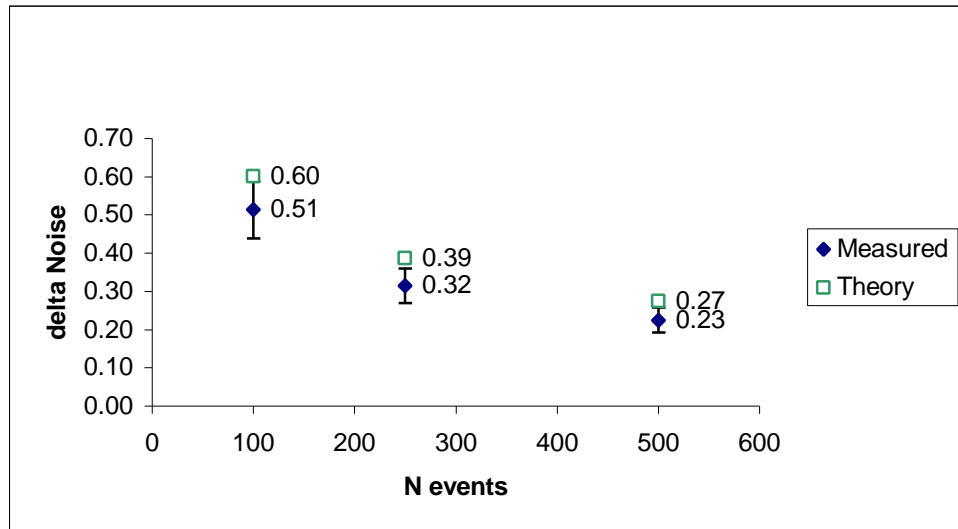


Figure 7. Empirical dependence of $\delta\sigma$ on N_{ev} (Measured) plotted with values predicted by the model (Theory). Each empirical data point has 25 contributing measurements. $\Delta V=7.5\text{mV}$ (default).

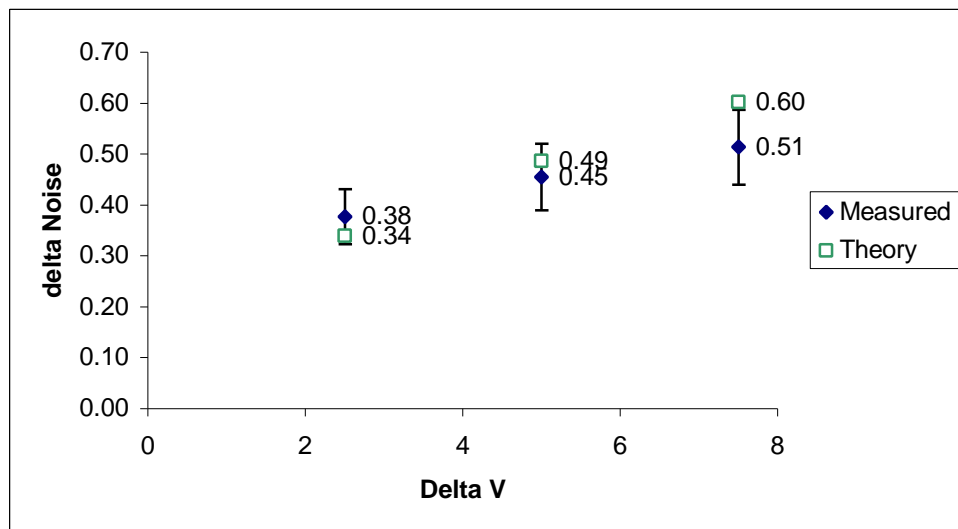


Figure 8. Empirical dependence of $\delta\sigma$ on ΔV (Measured) plotted with values predicted by the model (Theory). Each empirical data point has 25 contributing measurements. $N_{ev}=100$.

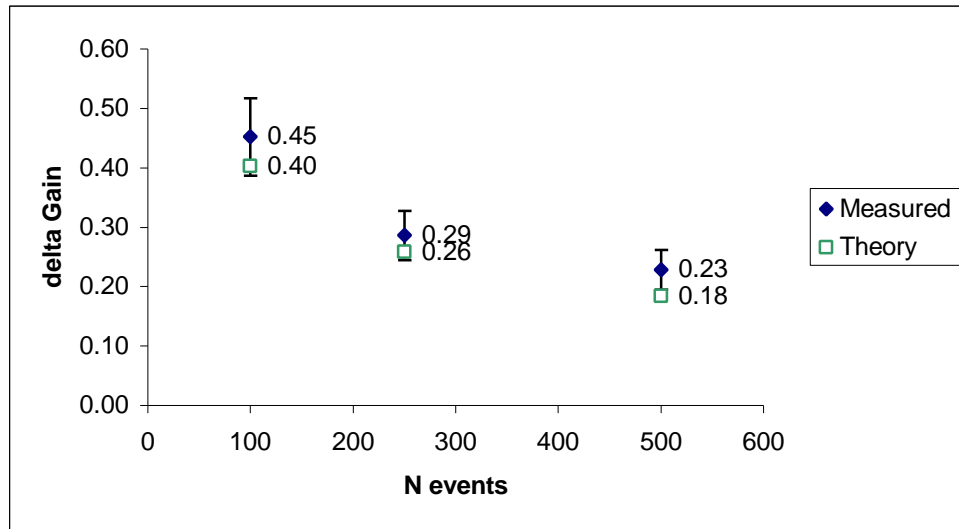


Figure 9. Empirical dependence of $\delta(\text{Gain})$ on N_{ev} (Measured) plotted with values predicted by the model (Theory). Each empirical data point has 25 contributing measurements. $\Delta V=7.5\text{mV}$ (default).

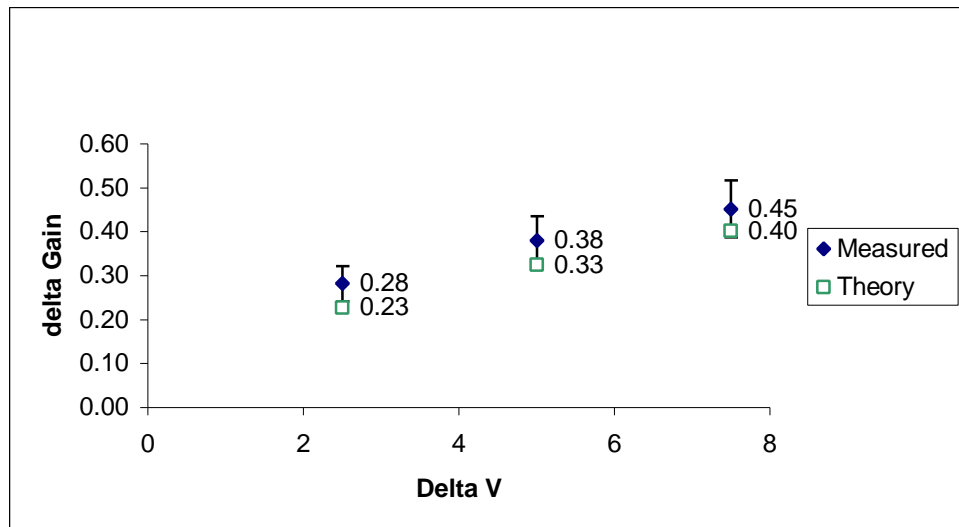


Figure 10. Empirical dependence of $\delta(\text{Gain})$ on ΔV (Measured) plotted with values predicted by the model (Theory). Each empirical data point has 25 contributing measurements. $N_{\text{ev}}=100$.

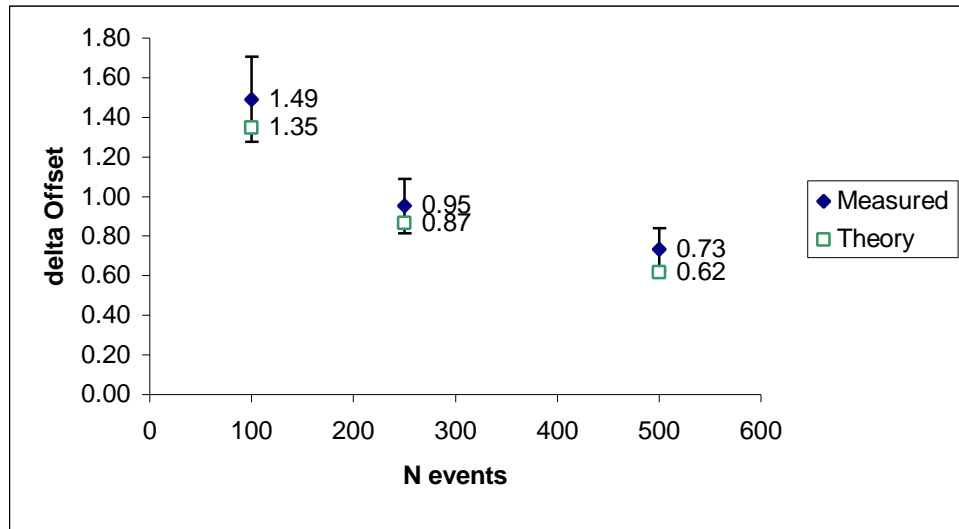


Figure 11. Empirical dependence of $\delta(\text{Offset})$ on N_{ev} (Measured) plotted with values predicted by the model (Theory). Each empirical data point has 25 contributing measurements. $\Delta V=7.5\text{mV}$ (default).

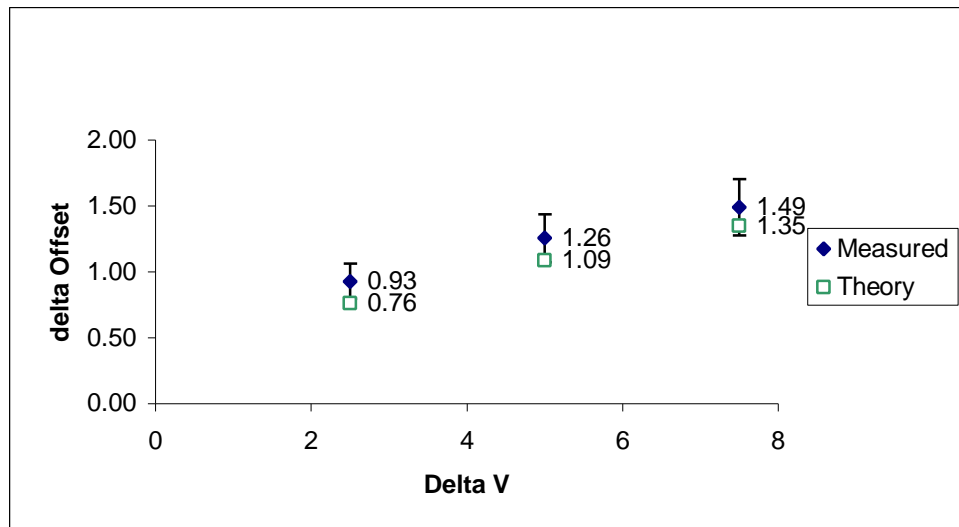


Figure 12. Empirical dependence of $\delta(\text{Offset})$ on ΔV (Measured) plotted with values predicted by the model (Theory). Each empirical data point has 25 contributing measurements. $N_{ev}=100$.

References.

- [1] ATLAS SCT Group, "Testing Specification for the Wafer Screening, Project: ABCD3T ASIC ", ATLAS Internal Document
- [2] ATLAS SCT Group, "Project Specification, Project: ABCD3T ASIC", ATLAS Internal Document
- [3] C. Haber et al, "Electrical Assembly and Testing of SVX Ladder Modules", CDF Note 1636, February 16, 1993
- [4] L. Lyons, A Practical Guide to Data Analysis for Physical Science Students, Cambridge University Press, c. 1991
- [5] Particle Data Group, R.M. Barnett et al., Physical Review **D54**, 1 (1996)
- [6] ROOT: An Object-Oriented Data Analysis Framework: <http://root.cern.ch>